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APPLICATION OF OFF-LINE AND ON-LINE IDENTIFICATION TECHNIQUES TO BUILDING SEISMIC RESPONSE DATA

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SUMMARY

The objectives of this paper are to present a comparison of the dynamic characteristics of a seven-storey reinforced concrete building (Van Nuys–Holiday Inn) identified from four recorded strong-motion response data (Whittier earthquake, Landers earthquake, Big Bear earthquake and Northridge earthquake). In the analysis, time-domain methods for estimating the system parameters and the modal properties of the building are studied. Both off-line and on-line identification algorithms are applied to these seismic response data. Under the assumption of a linear time-invariant system the ARX model and ARMAX model are used. Comparison of the identification results using different models are made. In addition, recursive procedures are adapted as on-line identification and the time-varying modal parameters are estimated. For structural systems under strong earthquake excitation, a recursive identification method, adaptive forgetting through multiple models (AFMM), is introduced to identify systems with rapidly changing parameters. Through the analysis of the seismic response data of the building subjected to four earthquakes the identification algorithm and the identification results are discussed.

KEY WORDS: seismic data processing; linear filtering theory; recursive identification method

INTRODUCTION

The aim of system identification is to find a mathematical model and to determine the modal parameters of a dynamic system using the excitation and response signals. There are different ways to relate the modal parameters to the measured data which leads to different identifying methods. Besides the identification methods, modal parameters can be identified either by a frequency domain method or by a time domain method. Most classical methods available in the field belong to the class of frequency domain methods. When using classical frequency domain identification methods, the identification of the modal parameters of a structure frequently fails in the case of closely spaced modal frequencies and especially in the presence of measurement noise. Over the past decades, new methods derived from automatics, such as time domain methods, have been successfully transferred to civil engineering applications.[1-7] Since all recordings are in discrete-time form, the discrete-time approach to identification is natural. A detailed analysis experimental data and the recorded seismic response of the building has been studied based on the discrete-time linear filtering approach with least-squares approximation.[8-10]

The purpose of this paper is to discuss the discrete-time method for system identification based on linear filtering and least-squares estimation. In the time domain identification method, some important and practical problems are discussed, such as the selection and validation of models in the identification. In addition, an unbiased adaptive recursive procedure was also adapted for modal identification problems. The modal parameters were estimated by using a recursive procedure with exponentially decaying weighting factors defined by the forgetting factor parameter from which the time-varying modal parameters can be estimated. A similar adaptive algorithm in system identification is the application of Kalman filtering

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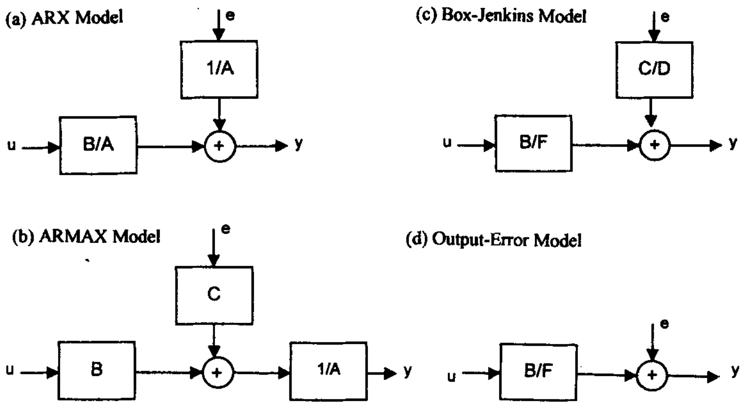


Figure 1. Block diagram for linear, discrete-time model: (a) ARX model; (b) ARMAX model; (c) Box–Jenkins model; (d) output-error model

techniques to identify the time-varying dynamic parameters of structures. Besides that a new recursive identification method suited for identification of systems with jumping or rapidly changing parameters is also introduced. It can be viewed as a method of implementing adaptive gains or adaptive forgetting factors in recursive identification. The method is called adaptive forgetting through multiple models (AFMM).[12] This method has been implemented in standard computer packages, MATLAB.[13] Application of this technique to the identification of the dynamic properties of the seismic response of damaged building is discussed.

MODELS OF LINEAR TIME-INVARIANT SYSTEMS

For the identification of the dynamic properties of building seismic response, two categories of time-series models are used in the discrete-time domain for linear time-invariant systems.

1. Equation error model structure

Consider a dynamic system with input $\{u(t)\}$ and output $\{y(t)\}$. The most simple input–output relationship is obtained by describing it as a linear difference equation:

$$y(t) + a_1 y(t-1) + \cdots + a_{n_a} y(t-n_a) = b_1 u(t-1) + \cdots + b_{n_b} u(t-n_b) + e(t) \tag{1}$$

where the white-noise term $e(t)$ here enters as a direct error in the difference equation. The adjustable parameters are in this case

$$\theta = [a_1, a_2, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}]^{\mathrm{T}} \tag{2}$$

If we introduce

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \\ B(q^{-1}) &= b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b} \end{aligned} \tag{3}$$

we shall have a model description:

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{1}{A(q^{-1})} e(t) \tag{4}$$

As shown in Figure 1(a), this model can be called an ARX model, where AR refers to the autoregressive part, $A(q^{-1})y(t)$, and X to the extra input, $B(q^{-1})u(t)$. Equation (1) can be rewritten as

$$y(t) = \theta^{\mathrm{T}} \psi(t) + e(t) \tag{5}$$

where

$$\psi^{T}(t) = (-y(t-1), \ldots, -y(t-n_a), u(t-1), \ldots, u(t-n_b))$$

This model describes the observed variable $y(t)$ as an unknown linear combination of the components of the observed vector $\psi(t)$ plus noise. Such a model is called a linear regression in statistics. In addition, one can also add flexibility to the model (as shown in equation (4)) by modelling the disturbance term $e(t)$, $e(t) = C(q^{-1})v(t)$. Then the resulting model is

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + \frac{C(q^{-1})}{A(q^{-1})}v(t) \tag{6}$$

where

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_{n_c} q^{-n_c}.$$

This is known as an ARMAX model, as shown in Figure 1(b).

2. Output error model structure (or transfer function model)

If we suppose that the relation between input and undisturbed output can be written as a linear difference equation, and that the disturbance consists of white measurement noise, then we obtain the following description:

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + e(t) \tag{7}$$

We call an output error (OE) model, as shown in Figure 1(d). A different form of the output error model, called the Box–Jenkins model, is used. It contains a noise term consisting of a white-noise sequence $\{e(t)\}$ filtered through the transfer function C/D:

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + \frac{C(q^{-1})}{D(q^{-1})}e(t) \tag{8}$$

An attractive property of the model is that it yields separate descriptions of the input–output relationship between u and y and the noise spectrum model described by C/D and $\{e(t)\}$, as shown in Figure 1(c).

From the discussion of the above models for linear time-invariant systems, it appears that the equation error model (ARX-model) is a linear estimation problem. One can compute the one-step-ahead prediction as

$$\hat{y}(t \mid \theta) = B(q^{-1})u(t) + [1 - A(q^{-1})]\,y(t) = \psi^{T}\theta \tag{9}$$

The prediction is a scalar product between a known data vector $\psi(t)$ and the parameter vector θ. Such a model is a linear regression in statistics. On the other hand, the output error method relies more on the accuracy of future output modelling. It is a non-linear estimation problem and may exhibit a local minimum. In this study, the maximum likelihood method is used to identify the model parameter of ARMAX model.

STRONG MOTION RECORDS FROM CSMIP Sta. #24386

The Van Nuys building is fairly regular in plan and represents a typical midheight non-ductile concrete moment frame building, as shown in Figure 2. The California Strong Motion Instrumentation Program (CSMIP) in the Department of Conservation, Division of Mines and Geology, collected four strong-motion records for this building:[14]

1. Whittier earthquake of 1 October 1987, $M_s = 6{\cdot}1$
2. Landers earthquake of 28 June 1992, $M_s = 7{\cdot}6$
3. Big Bear earthquake of 28 June 1992, $M_s = 6{\cdot}6$
4. Northridge earthquake of 17 January 1994, $M_s = 6{\cdot}8$

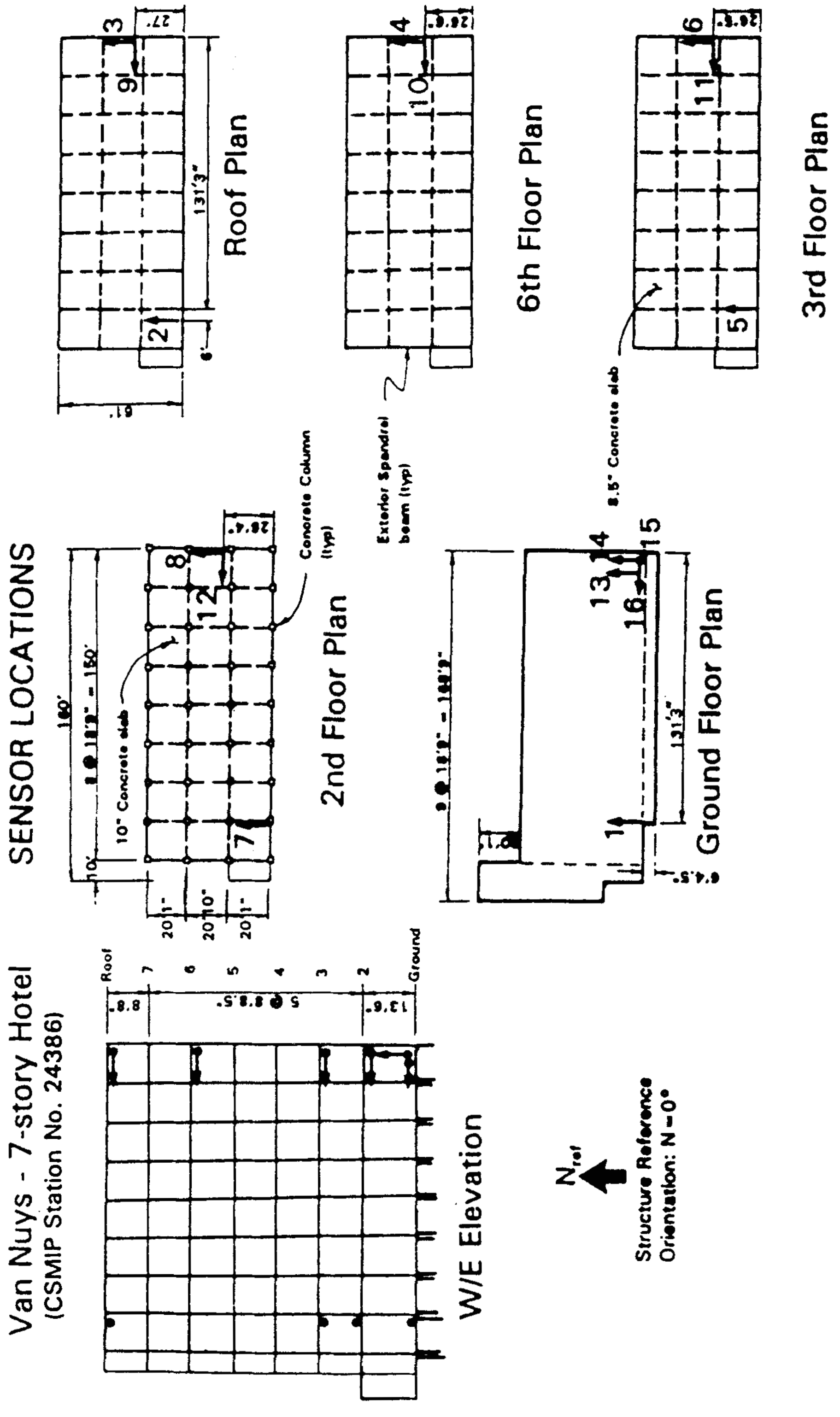


Figure 2. Plan view and sensor locations of the Van Nuys seven-storey building (CSMIP Station: 24386)

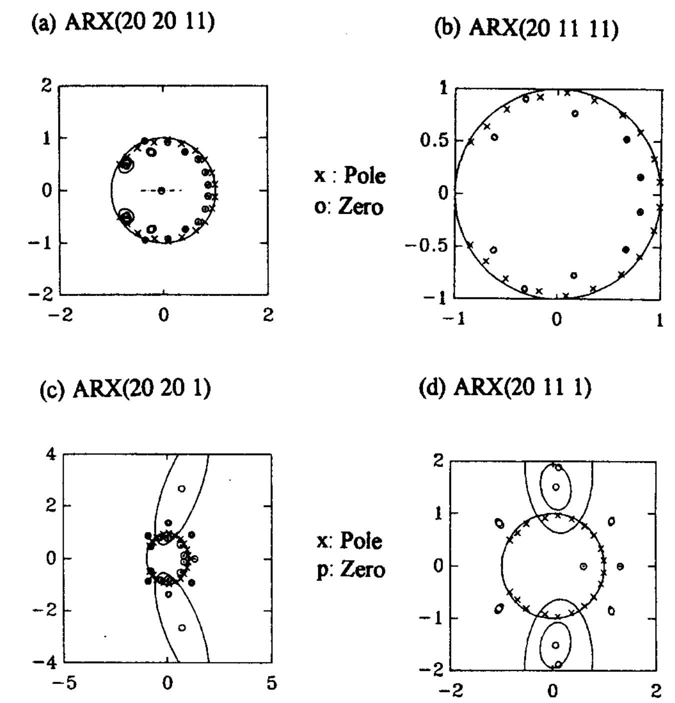


Figure 3. Comparison on the pole-zero locations and confidence regions by using different model order and time delay between input and output: (a) ARX (20, 20, 11); (b) ARX (20, 11, 11); (c) ARX (20, 20, 1); (d) ARX (20, 11,1)

The strong-motion records available for this research represent low to strong input earthquake motions. The input base accelerations ranged from a maximum of 408·9 to a low of 22·3 cm/sec^2. Damage was observed in this building for the Northridge earthquake. In the present study only the data recorded in the north south direction are used for identification.

RESULTS FROM OFF-LINE IDENTIFICATION

Both ARX and ARMAX models are used for which the model parameters are identified. For each single-input and single-output pair, we identify the filter by completing the following steps.

1. Determine the time delay between input and output: In order to check whether the system has feedback phenomenon (such as soil–structure interaction), it is necessary to determine the exact time delay between input and output signals.
2. Select the model order, calculate the filter parameters and check the pole-zero cancellation in the filter: Reducing the order in $B(q^{-1})$ may help to avoid the pole-zero cancellation, but it will increase the error index ($\sum(y-\hat{y})^2/\sum y^2$). In order to reduce the error, one may increase the order in $A(q^{-1})$ and $B(q^{-1})$.
3. Check the autocorrelation of residuals for whiteness and the cross correlation of residuals with input for independence. The residual is defined as the difference between the recorded output and the output from the identified model.
4. If Steps 2 and 3 are satisfied, calculate the transfer function, modal frequencies and damping ratios of the system.

Figure 3 plots the poles and zeros of ARX (n_a, n_b,d) model with different model order. Time-delay steps between input and output can be specified as 'd-steps' in the ARX model. It is found that if the time delay between input and output is considered, the poles and zeros are all within a unit circle (i.e. a stable system is

Table I. Comparison on the estimated model parameter of $B(q^{-1})$ with and without considering time delay between input and output

(20 11 11)	$b1$	$b2$	$b3$	$b4$	$b5$	$b6$	$b7$	$b8$	$b9$	$b10$	$b11$
$B(q^{-1})$	0·5449	−0·7558	−0·7467	−0·8949	0·8545	−0·6101	0·4649	−0·3831	−0·2931	−0·2250	0·0996
σ	0·0150	0·0397	0·0600	0·0723	0·0800	0·0833	0·0818	0·0751	0·0636	0·0442	0·0178
(20 11 1)	$b1$	$b2$	$b3$	$b4$	$b5$	$b6$	$b7$	$b8$	$b9$	$b10$	$b11$
$B(q^{-1})$	−0·0151	0·0366	−0·1013	0·1696	−0·1898	0·1954	−0·2750	0·3916	−0·6234	0·8479	−0·3434
σ	0·0147	0·0381	0·0579	0·0702	0·0767	0·0789	0·0782	0·0731	0·0618	0·0418	0·0165

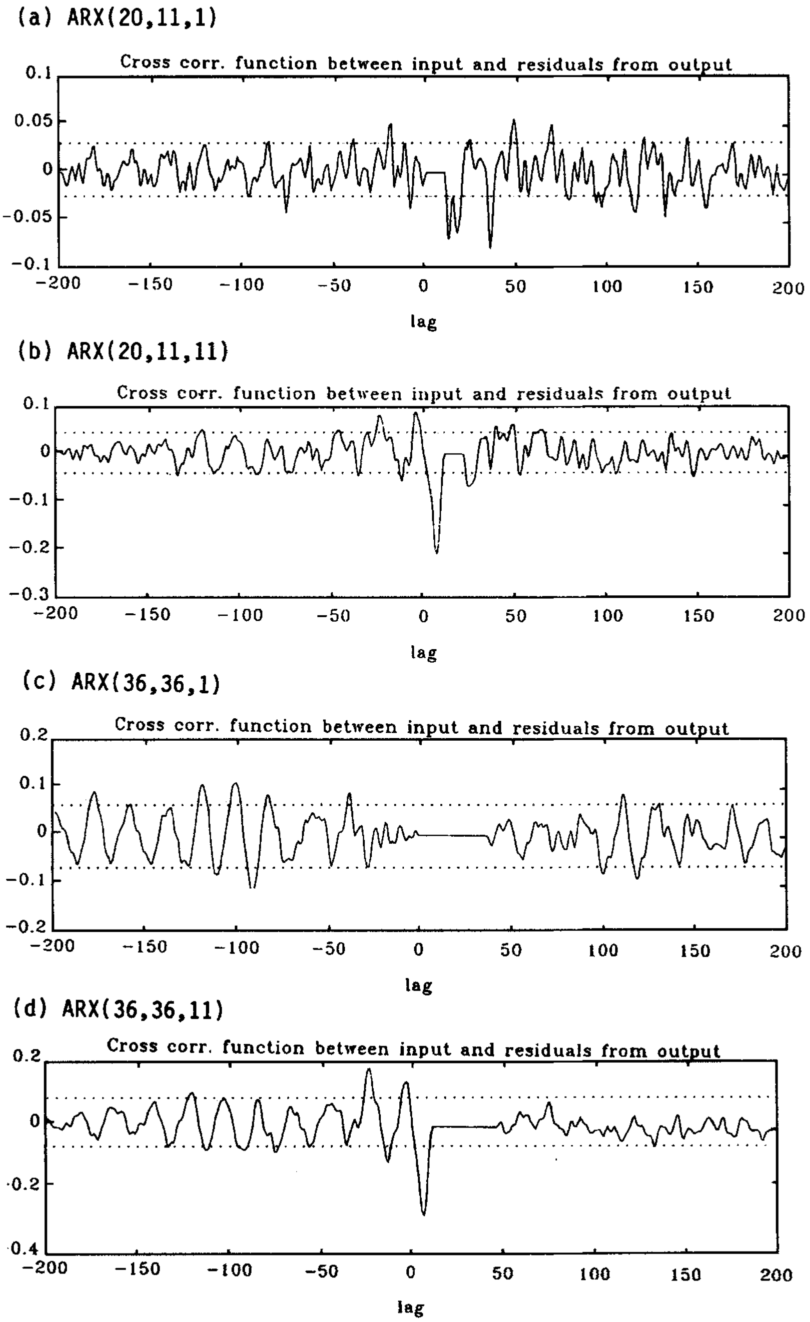


Figure 4. Cross-correlation functions between input and residuals from output by using different model order and time delay: (a) ARX (20, 11, 1); (b) ARX (20, 11, 11); (c) ARX (36, 36, 1); (d) ARX (36, 36, 11)

identified). It means that with a suitable reduction of the order n_b, the pole-zero cancellation can be avoided. It should also be pointed out that the standard deviation of the estimated model parameters in $B(q)$ is much reduced if the time delay between input and output is considered, as shown in Table I. The cross covariance of residuals with input is also influenced by the effect of time delay as shown in Figure 4. For positive correlation lags, the cross covariance shows whether more information can be extracted from residuals by the

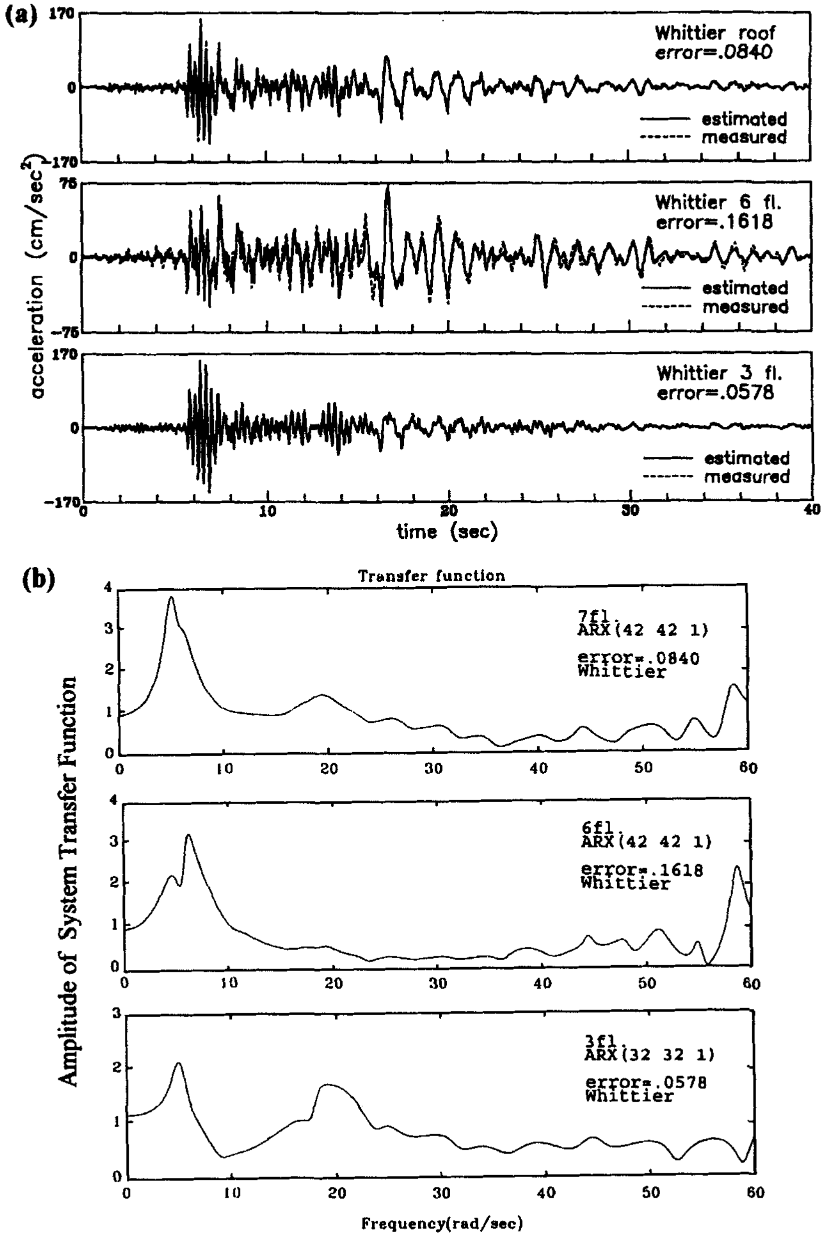


Figure 5. Results of identification from 1987 Whittier earthquake using ARX model: (a) comparison on the recorded and the estimated floor acceleration; (b) estimated transfer functions

input. For negative correlation lags, the cross covariance shows if there is a feedback in the system. The cross covariance is also given in Figure 4 with 99 per cent confidence levels. For systems for which time-delay effect is considered the confidence criterion is satisfied for both positive lag and negative lag.

Following the above-mentioned procedures, analyses were performed on the data of the Van Nuys building. Under the assumption of a linear system, both ARX and ARMAX models were applied. It must be pointed out that increasing the order of the ARX model can reduce the prediction error and under such circumstance both time-delay and pole-zero cancellation are not important issues in the analysis, and characteristics of close modes can also be identified by increasing the order of the ARX model. As shown in Figures 5 and 6,

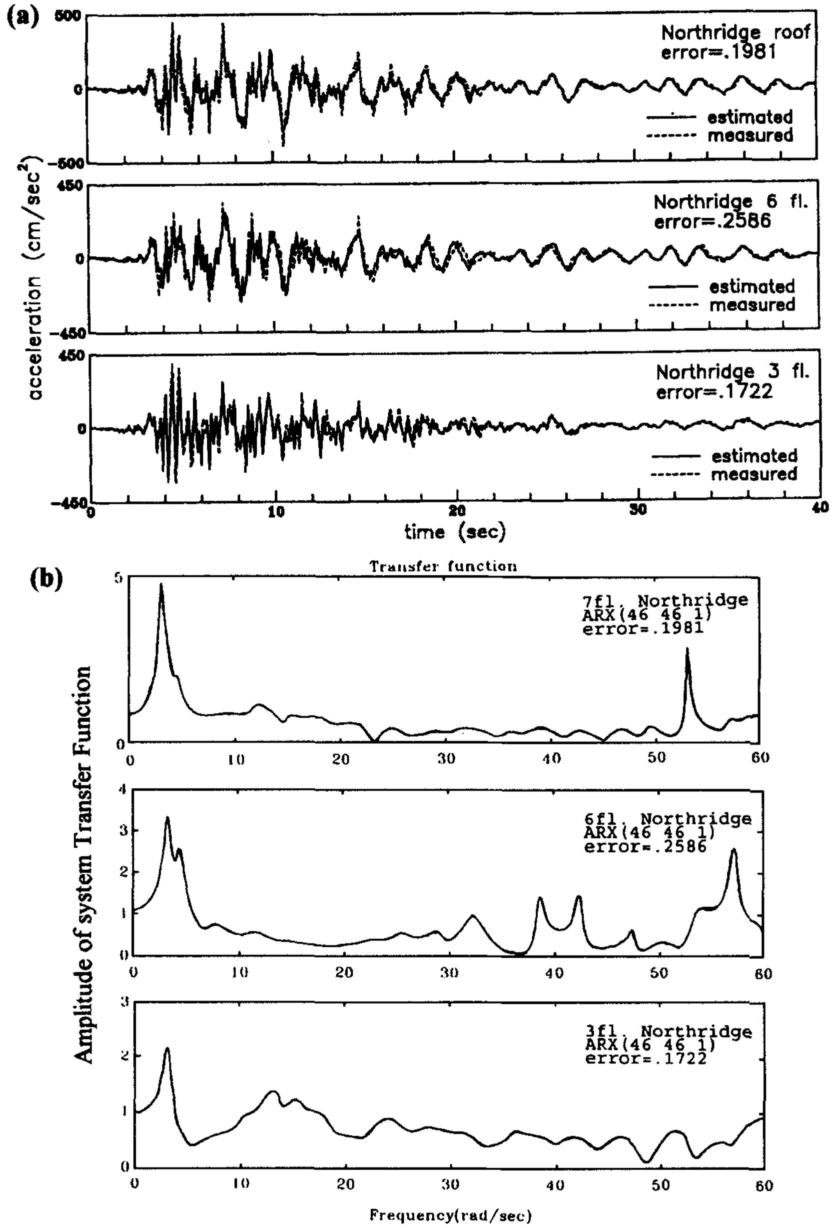


Figure 6. Results of identification from 1994 Northridge earthquake using ARX model: (a) comparison on the recorded and the estimated floor accelerations; (b) estimated transfer function

the close modes can be identified from the response data. This phenomenon is observed more clearly in the identified transfer function of the 6th floor. (Since only a single component of motion was collected from the response of the 6th floor, the torsional motion cannot be removed from the data). On the contrary, the ARMAX model cannot identify the closely spaced modes but it provided a smooth transfer function, from which it is easy to construct the mode shape of the building. Figures 5 and 6 also show the estimated transfer functions and the comparison on the predicted response to the recorded response from the data of Whittier and Northridge earthquakes. The estimated modal frequencies, damping ratios, modal contributions, and phase angles are shown in Tables II and III for the two earthquakes, respectively. *It should be emphasized*

Table II. Comparison between ARX model and ARMAX model on the estimated modal parameters by using data from Whittier earthquake

Frequency (rad/sec)	Damping ratio	Modal contribution	Phase angle (degree)	Pole magnitude
Roof, ARX(42 42 1), Error = 0·0840				
5·1468	0·1411	0·0855	89·8208	0·9714
6·1506	0·1467	0·0381	105·8051	0·9646
13·8977	0·3293	0·009	−62·753	0·8327
19·2638	0·1142	0·0068	−39·7514	0·9158
6th floor, ARX(42 42 1), Error = 0·1618				
5·0643	0·2824	0·1343	101·7916	0·9444
8·8449	0·2984	0·0356	−31·7182	0·8998
5·9086	0·0887	0·0338	−171·799	0·9793
11·0739	0·1255	0·0028	90·5513	0·9459
3rd floor, ARX(42 42 1), Error = 0·0578				
5·1225	0·1622	0·0418	113·8056	0·9673
15·1276	0·3511	0·0193	−101·312	0·8086
37.5326	0·2416	0·0176	44·3315	0·6958
22·3083	0·1779	0·0159	28·0477	0·8532
Roof, ARMAX(12 12 11 1), Error = 0·0921				
5·4566	0·1749	0·1282	90·6622	0·9626
19·7598	0·1327	0·0081	−75·025	0·9004
30·2289	0·1842	0·0034	−179·338	0·8003
60·709	0·0425	0·0012	−89·8621	0·9019
6th floor, ARMAX(12 12 11 1), Error = 0·1946				
6·5272	0·2045	0·1022	79·8987	0·948
64·4792	0·0834	0·0019	−135·672	0·8065
48·0102	0·0752	0·0016	−158·207	0·8655
60·2585	0·0407	0·0009	94·9133	0·9065
3rd floor, ARMAX(12 12 11 1), Error = 0·0581				
5·5924	0·1951	0·0566	86·7989	0·9573
19·8965	0·1334	0·01	116·4413	0·8993
62·4702	0·2334	0·008	−4·6906	0·5581
57·0322	0·1119	0·0015	172·4852	0·7746

Note: The modal frequency, damping and phase angle for each mode are calculated based on the formula in Reference 1

that the present off-line identification can only represent the system in its equivalent linear state (see Figure 7 for the identified mode shape). If the system was damaged during strong shaking (such as the data of Northridge earthquake), the time-varying modal parameters cannot be identified from off-line identification. A different approach to this problem will be discussed in the next section.

RECURSIVE IDENTIFICATION ALGORITHM

During strong earthquake excitation, a building structure may behave as a non-linear system. The system modal parameters may change with respect to time during the excitation. It is called a time-variant system. The above-mentioned algorithm of off-line identification can only be applied to the time-invariant system. It is necessary to use a recursive identification algorithm to identify the time variation of the model parameters. The following two methods are introduced in this study with respect to the ARX model on the identification of the seismic response data of the Van Nuys building.

Table III. Comparison between ARX model and ARMAX model on the estimated modal parameters by using data from Northridge earthquake

Frequency (rad/sec)	Damping ratio	Modal contribution	Phase angle (degree)	Pole magnitude
Roof, ARX(46 46 1), Error = 0·1981				
3·1575	0·1125	0·1587	97·574	0·9859
10·4483	0·2403	0·0227	−67·2459	0·9044
4·6446	0·1114	0·0198	34·3652	0·9795
33·6291	0·3078	0·0134	153·1274	0·6609
6th floor, ARX(46 46 1), Error = 0·2586				
3·3946	0·1368	0·1046	88·2859	0·9816
4·4967	0·1075	0·0433	69·8765	0·9809
7·6375	0·1288	0·0067	55·2224	0·9614
11·2491	0·1007	0·0024	13·5425	0·9557
3rd floor, ARX(46 46 1), Error = 0·1722				
3·2218	0·1491	0·0769	102·3774	0·981
34·1745	0·5689	0·0135	13·9745	0·4595
13·2976	0·0821	0·0053	93·6878	0·9572
5·2569	0·1664	0·0048	−43·7886	0·9656
Roof, ARMAX(20 20 15 1), Error = 0·2345				
3·0919	0·1538	0·2237	95·2493	0·9812
18·6721	0·1704	0·0082	−159·081	0·8805
12·5378	0·0919	0·0076	−105·36	0·955
36·0941	0·0949	0·002	−27·4388	0·872
6th floor, ARMAX(20 20 19 1), Error = 0·3189				
3·6967	0·1646	0·1567	76·8184	0·9759
11·5392	0·2102	0·0066	−28·1532	0·9075
23·2251	0·1016	0·0018	−78·0762	0·91
32·6839	0·0207	0·0008	115·5141	0·9734
3rd floor, ARMAX(20 20 15 1). Error = 0·1827				
3·2003	0·1399	0·0803	95·5413	0·9823
12·8195	0·1476	0·0103	97·456	0·9271
36·2093	0·2176	0·0046	−16·3369	0·7297
16·2668	0·0876	0·0032	73·6746	0·9446

Note: The modal frequency, damping and phase angle for each mode are calculated based on the formula in Reference 1

Recursive least-square method (RLS)

A different approach for parameter estimation is the recursive (on-line) identification method. The parameter estimates are computed recursively in time. Consider the Equation Error Model (ARX-model), and the parameter estimate is given by

$$\hat{\theta} = \left[\sum_{s=1}^{t} \psi(s)\psi^{\mathrm{T}}(s) \right]^{-1} \left[\sum_{s=1}^{t} \psi(s)y(s) \right] \tag{10}$$

Introduce the notation

$$P(t) = \left[\sum_{s=1}^{t} \psi(s)\psi^{\mathrm{T}}(s) \right]^{-1} \tag{11}$$

Since trivially

$$P^{-1}(t) = P^{-1}(t-1) + \psi(t)\psi^{\mathrm{T}}(t) \tag{12}$$

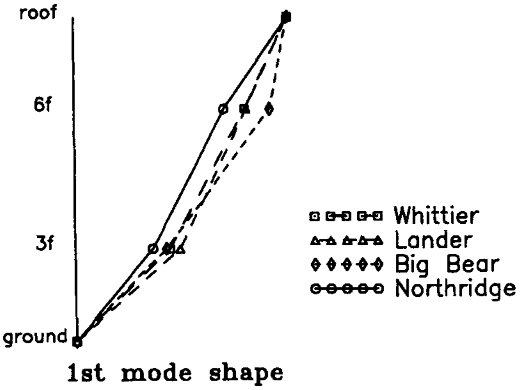


Figure 7. Comparison on the estimated mode shape (Ist mode) of the Van Nuys Building from four different earthquake data

it follows that

$$\begin{aligned}\hat{\theta}(t) &= P(t)\left[\sum_{s=1}^{t-1} \psi(s)y(s) + \psi(t)y(t)\right] \\ &= P(t)[P^{-1}(t-1)\hat{\theta}(t-1) + \psi(t)y(t)] \\ &= \hat{\theta}(t-1) + P(t)\psi(t)[y(t) - \psi^{\mathrm{T}}(t)\hat{\theta}(t-1)] \end{aligned} \tag{13}$$

Let $\varepsilon(t) = y(t) - \psi^{\mathrm{T}}(t)\hat{\theta}(t-1)$, and it is the difference between the measured output $y(t)$ and the one-step-ahead prediction $\hat{y}[t|t-1; \hat{\theta}(t-1)] = \psi^{\mathrm{T}}(t)\hat{\theta}(t-1)$ of $y(t)$ made at time $t-1$ based on the model corresponding to the estimate $\hat{\theta}(t-1)$. Using the matrix inversion lemma, the updating equation for $P(t)$ is obtained, namely

$$P(t) = P(t-1) - P(t-1)\psi(t)\psi^{\mathrm{T}}(t)P(t-1)/[1 + \psi^{\mathrm{T}}(t)P(t-1)\psi(t)] \tag{14}$$

It is now a scalar division instead of a matrix inversion. If the dynamic properties of the structural system change (slowly) with time, the recursive algorithm should be able to track the time-varying parameters describing such a system. One of the approaches in this case is to change the objective function to be minimized. Let the modified objective function be

$$V_t(\theta) = \sum_{s=1}^{t} \beta(t, s)[y(s) - \psi^{\mathrm{T}}\hat{\theta}(s-1)]^2 \tag{15}$$

where

$$\beta(t, s) = \left[\prod_{j=s}^{t-1} \lambda(j)\right] \quad \text{and} \quad \beta(t, t) = 1{\cdot}0$$

It contains the forgetting factor λ, a number somewhat less than 1. This means that with increasing t the measurements obtained previously are discounted. The smaller the value of λ, the quicker the information in previous data will be forgotten. In practice, it has often been useful to let $\lambda(t)$ grow exponentially with t to 1. This can be written as

$$\lambda(t) = \lambda_0\lambda(t-1) + (1 - \lambda_0) \tag{16}$$

where the rate λ_0 and the initial value $\lambda(0)$ are design variables. Figure 8(a) shows the $\lambda(t)$ as a function of sample point for different values of $\lambda(0)$ and λ_0. Once both $\lambda(0)$ and λ_0 are set, the shape of $\lambda(t)$ is fixed. The corresponding weighting profile $\beta(t, s)$ is also shown in Figure 8(b). It shows that at the beginning of the data sequence (small number of sample points), the initial condition may have influence on the accuracy of the

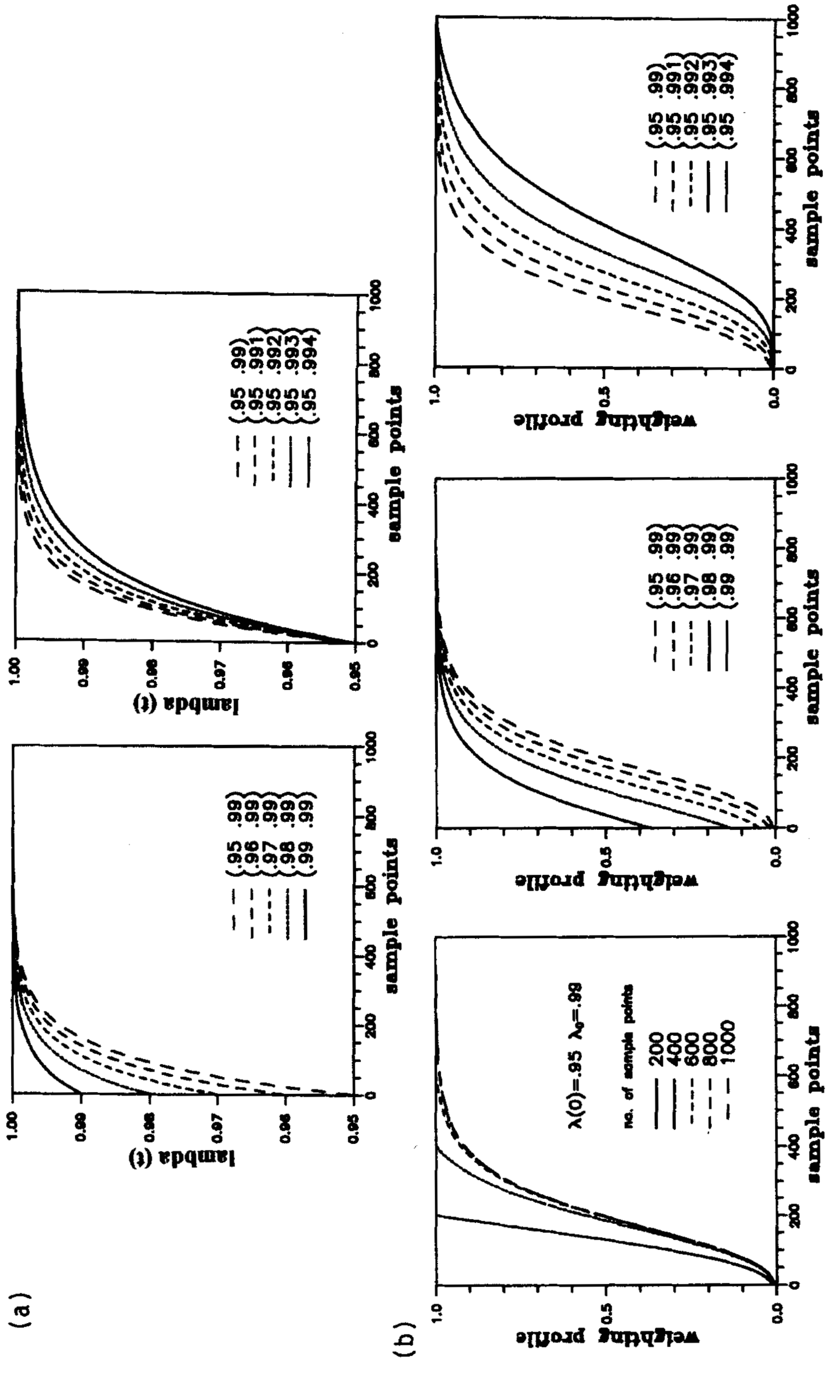


Figure 8. (a) Time-dependent forgetting factor $\lambda(t)$ with different modal parameters $(\lambda(0), \lambda_0)$. (b) The weighting profile of $\beta(t, s)$ which corresponds to each $(\lambda(0), \lambda_0)$

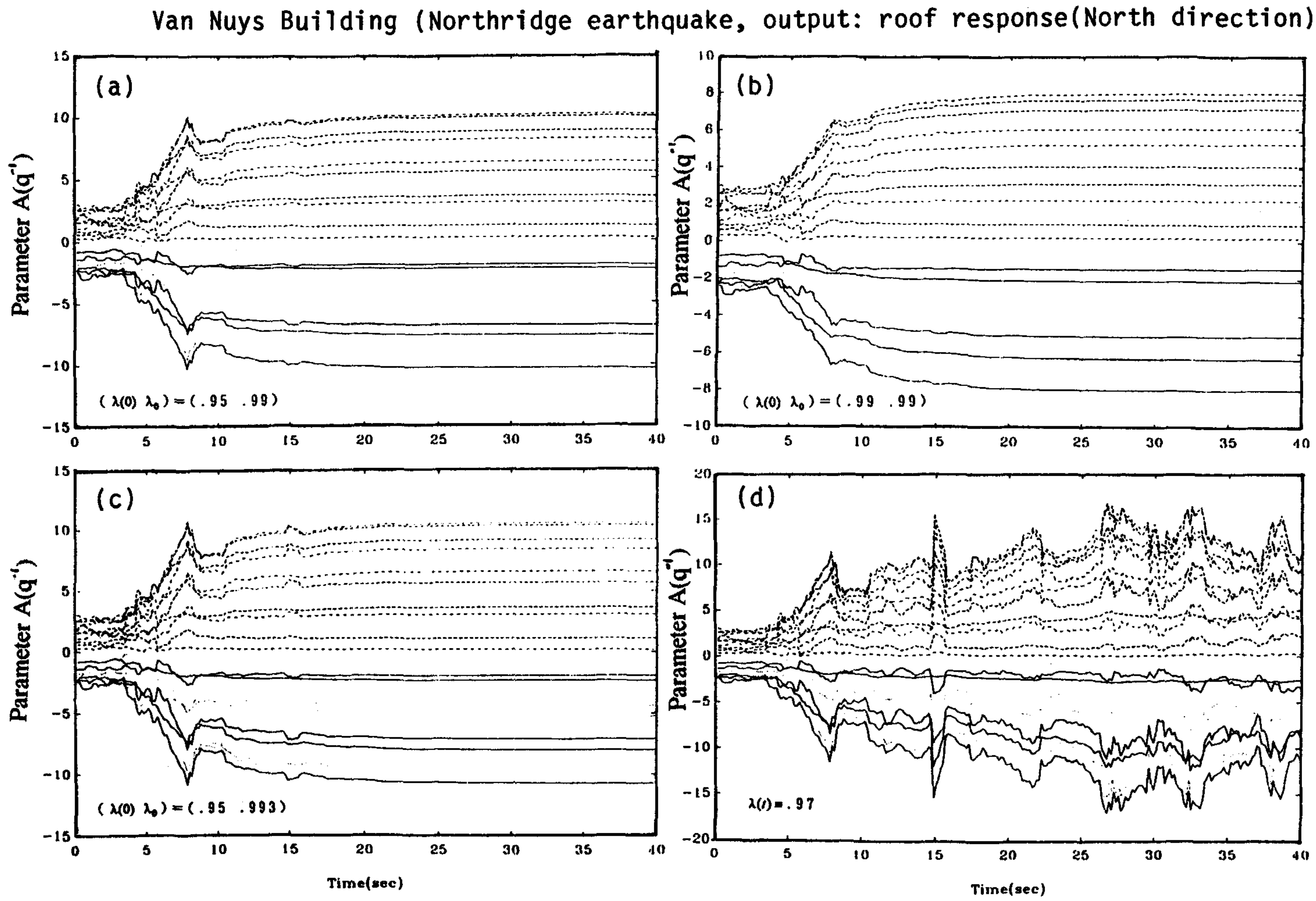


Figure 9. Time variation of model parameter $A(q)$ by using RLS method with different initial value of forgetting factor: (a) $(\lambda(0), \lambda_0) = (0{\cdot}95, 0{\cdot}99)$; (b) $(\lambda(0), \lambda_0) = (0{\cdot}99, 0{\cdot}99)$; (c) $(\lambda(0), \lambda_0) = (0{\cdot}95, 0{\cdot}993)$; (d) $\lambda(t) = 0{\cdot}97$

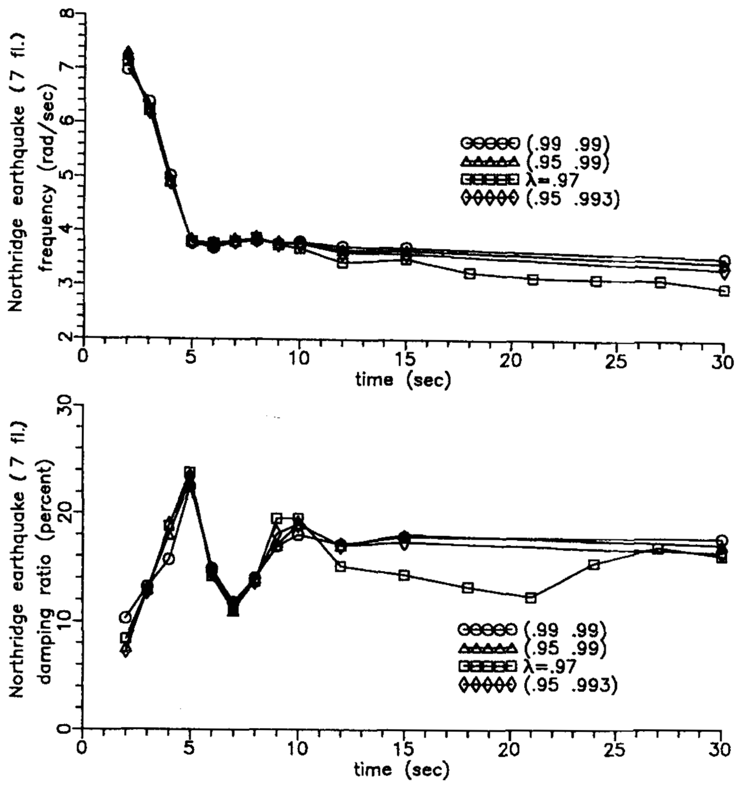


Figure 10. Estimated time variation of natural frequency and damping ratio of the Ist mode of the Van Nuys building from Northridge earthquake by using ifferent parameters of forgetting factor

estimates so that the weighting should be small for data away from the present state. Finally, with this forgetting factor the modified RLS algorithm can be expressed as

$$
\begin{aligned}
\hat{\theta}(t) &= \hat{\theta}(t-1) + K(t)\varepsilon(t) \\
\varepsilon(t) &= y(t) - \psi^{\mathrm{T}}(t)\hat{\theta}(t-1) \\
K(t) &= P(t)\psi(t) = P(t-1)\psi(t)/[\lambda(t) + \psi^{\mathrm{T}}(t)P(t-1)\psi(t)] \\
P(t) &= \frac{1}{\lambda(t)}\{P(t-1) - P(t-1)\psi(t)\psi^{\mathrm{T}}(t)P(t-1)/[\lambda(t) + \psi^{\mathrm{T}}(t)P(t-1)\psi(t)]\}
\end{aligned}
\tag{17}
$$

Results from RLS method

Based on the response data from the Northridge earthquake, the RLS method was applied. An ARX model with orders 20 and 11 for the polynomials $A(q^{-1})$ and $B(q^{-1})$ was selected. Since it was known that the building was damaged during the earthquake, forgetting factors with differential initial values were used in the algorithm to observe the time variation of the parameter $A(q^{-1})$, as shown in Figure 9. Different values of $\lambda(0)$ and λ_0 may give different results for the estimation of the modal parameters. The values of the corresponding frequencies and damping ratios for different parameters of forgetting factors are shown in Figure 10. With the same technique, Figure 11 shows the estimated natural frequency and damping ratio of the first mode of the Van Nuys building from seismic data for four different earthquakes. The order of the ARX model and the parameters of forgetting factor for each data set of seismic events is indicated in the figure. The specified time interval in the figure denotes the first initial time segment used for the off-line identification so as to estimate the modal parameters as initial values for the on-line identification.

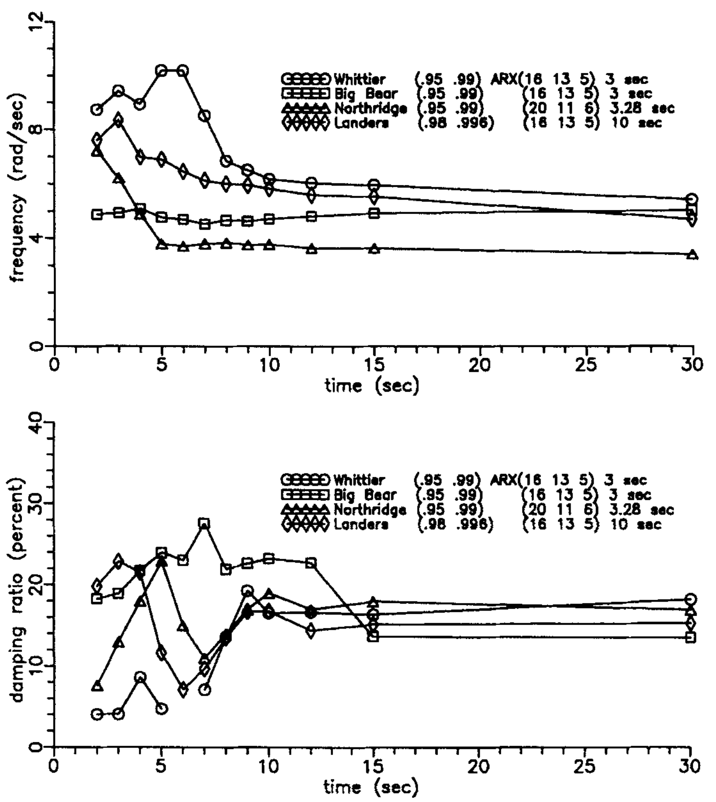


Figure 11. Comparison on the estimated natural frequency and damping ratio of the Van Nuys building from four different earthquake data

There are some unreasonable results in Figure 11. First, the identified natural frequencies from each seismic event to another seismic event are not consistent. That is, the identified natural frequency at the end of Whittier earthquake (1 October 1987) does not coincide with the identified natural frequency from the Big Bear or Lander earthquake (28 June 1992). Second, the time variation of the identified damping ratio during the first 10 sec of the record changes too rapidly. The reason is that the forgetting factor is not adaptive and cannot reflect the abrupt change of modal parameters. It is necessary to find a method that can identify systems with rapidly changing parameters, or to use an adaptive forgetting factor for recursive identification.

Adaptive forgetting through multiple models

A new recursive identification method, adaptive forgetting through multiple model (AFMM), which is capable of identifying systems with changing parameters has been proposed by Anderson.[12] This method implements the notion of adaptation of the forgetting factor in a recursive identification algorithm so that both slow, fast and abrupt changes in dynamics may be tracked adequately.

One possible description of a discrete-time system with jumping parameters is the state model

$$\begin{aligned} \theta(t+1) &= \theta(t) + w(t) \\ y(t) &= \varphi^{\mathrm{T}}(t)\theta(t) + e(t) \end{aligned} \tag{18}$$

where $\theta(t)$ is an n-dimensional vector containing the time parameters describing the system at time t, $\varphi(t)$ is a vector containing old inputs and outputs, and $e(t)$ and $w(t)$ are disturbances with variance R_2 and R_1, respectively. If $w(t)$ is not Gaussian, one has a non-linear filtering problem. The Kalman filter does not provide the optimal solution. Here one shall pursue an approach based on finite-Gaussian sum

approximation. Suppose that the posterior distribution of $\theta(t)$ given y^{t-1} (all old y's up to and include $y(t-1)$) can be approximated with a sum of M Gaussian density functions. The posterior density function for $\theta(t)$ can be written

$$p[\theta(t)|y^{t-1}] = \sum_{i=1}^{M} \alpha_i(t) G_n(\theta(t), \bar{\theta}_i(t), P_i(t)) \tag{19}$$

where $\sum \alpha_i(t) = 1$ and $\bar{\theta}_i(t)$ and $P_i(t)$ are the mean vectors and covariance matrices, respectively, of the different Gaussian distributions at time t. Using Bayes' rule, one can use $p(\theta(t)|y^t)$ to compute $p(\theta(t-1)|y^t)$. Detail derivations can be seen from Reference 12. From the derivation we see that each Gaussian distribution splits into two. The approximation is to cut off M branches and let only the most likely component branch, and also cut off the least likely component entirely. It gives the algorithm of the AFMM method which can be summarized as follows:[12]

For $i = 1, 2, \ldots, M$,

$$\begin{aligned} P_i(t) &= P_i(t-1) - \frac{P_i(t-1)\varphi(t)\varphi^{\mathrm{T}}(t)P_i(t-1)}{R_2 + \varphi^{\mathrm{T}}(t)P_i(t-1)\varphi(t)} \\ \varepsilon_i(t) &= y(t) - \varphi^{\mathrm{T}}(t)\bar{\theta}_i(t-1) \\ \bar{\theta}_i(t) &= \bar{\theta}_i(t-1) + \frac{1}{R_2} P_i(t)\, \varphi(t)\varepsilon_i(t) \\ \bar{\alpha}_i(t) &= \frac{\alpha_i(t)}{\sqrt{(R_2 + \varphi^{\mathrm{T}}(t)P_i(t-1)\varphi(t)}} \exp\left(-\frac{1}{2}\frac{\varepsilon_i^2(t)}{R_2 + \varphi^{\mathrm{T}}(t)P_i(t-1)\varphi(t)} \right) \end{aligned} \tag{20}$$

$$\left. \begin{aligned} i_{\min} &= \arg\min \bar{\alpha}_i(t) \\ i_{\max} &= \arg\max \bar{\alpha}_i(t) \\ P_{i\min}(t) &= R_1 + P_{i\max}(t) \end{aligned} \right\} \tag{20a}$$

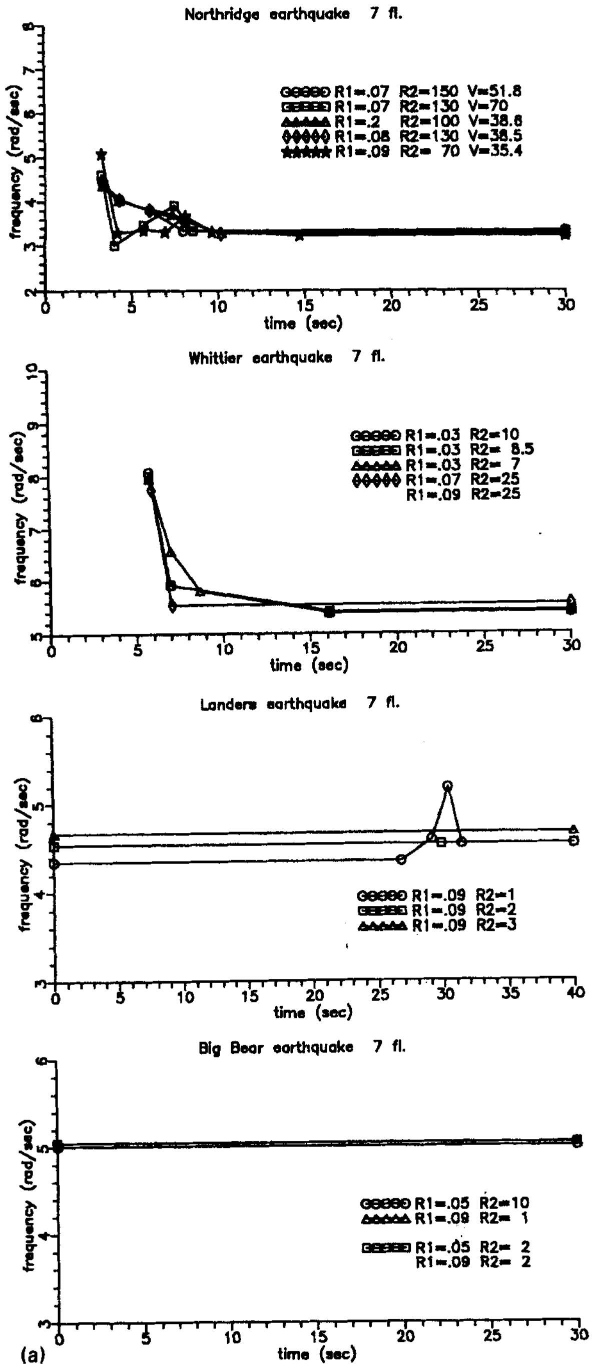
$$\left. \begin{aligned} \bar{\theta}_{i\min} &= \bar{\theta}_{i\max}(t) \\ \bar{\alpha}_{i\min}(t) &= q \cdot \bar{\alpha}_{i\max}(t) \\ \alpha_i(t) &= \frac{1}{\sum_{k=1}^{M} \bar{\alpha}_k(t)} \bar{\alpha}_i(t) \end{aligned} \right\} \tag{20b}$$

The estimation $\hat{\theta}(t)$, of $\theta(t)$ then becomes

$$\hat{\theta}(t) = \sum_{i=1}^{M} \alpha_i(t)\bar{\theta}_i(t) \tag{21}$$

Results from AFMM algorithm

To test the AFMM algorithm, we select an ARX(20, 11, 6) Model and apply it to the same data collected from the Van Nuys building during these four earthquakes. The initial values of $\bar{\theta}_i(t)$ and $P_i(0)$ are evaluated from the least-squares method for the initial motion (the first 3·28 sec from the records), and α_i is set to $1/M$ (where M is one of the design parameters, $M = 10$, in this study), and q is set equal to 10^{-5}. The most important design variables are R_1 and R_2 (the results of the analysis are not sensitive to M and q). Figure 12 shows the time variation of the identified system natural frequency and damping ratio for different sets of R_1 and R_2 values. Because the implement of the adaptive gain for each set of seismic data is different, the changes in system dynamics can be clearly identified. It depends on the time variation of the system characteristics. The choice of R_2 is probably more critical. The sensitivity with respect to R_2 can be a serious problem if it is difficult to estimate this in advance during the identification. Figure 12 also provides the sensitivity study with respect to both R_1 and R_2 on the estimation of modal parameters. It is clearly observed from Figures 12(a) and 12(b) that the data from the Northridge earthquake show more changes in system



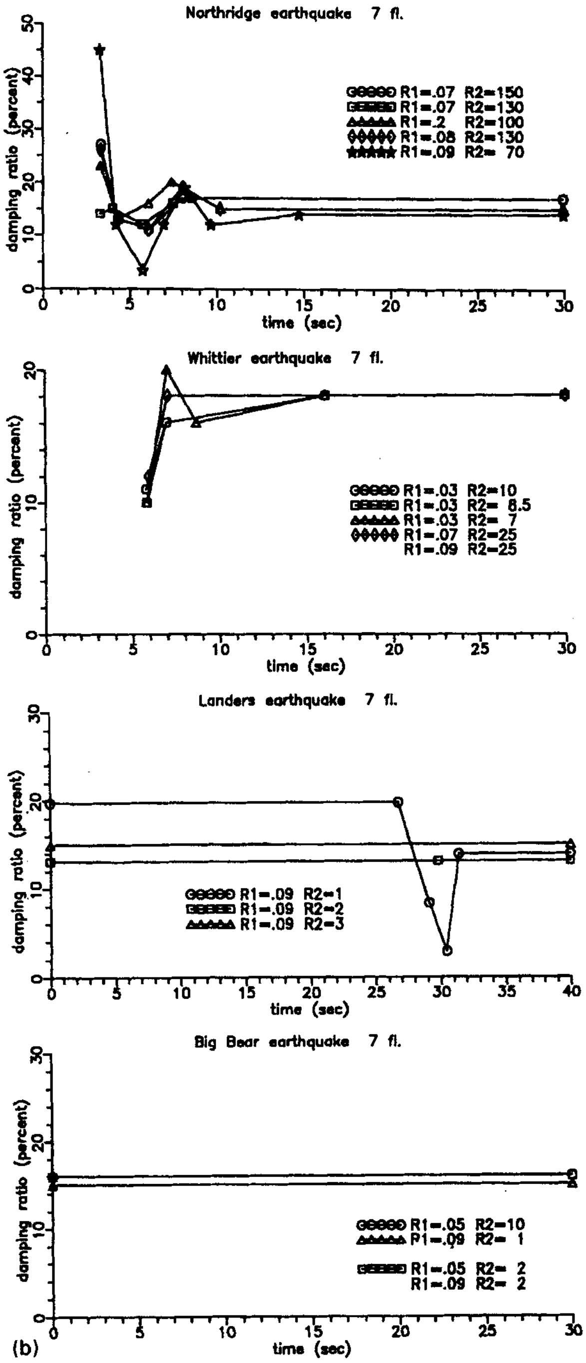
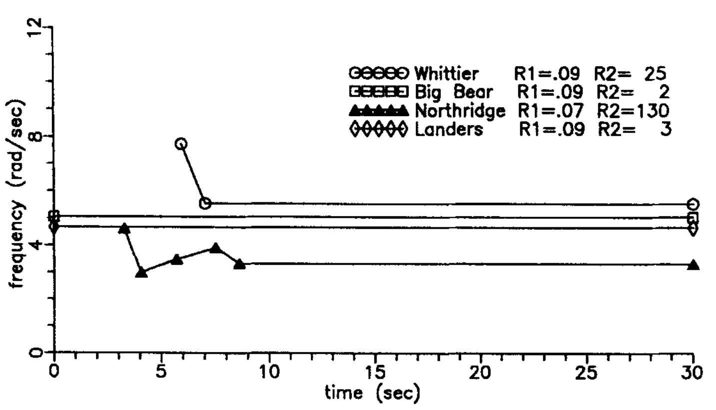


Figure 12(a). Identified time-variation natural frequency (fundamental mode) of the Van Nuys building from four different seismic response data using AFMM algorithm

Figure 12(b). Identified time-variation damping ratio (fundamental mode) of the Van Nuys building from four different seismic response data using AFMM algorithm



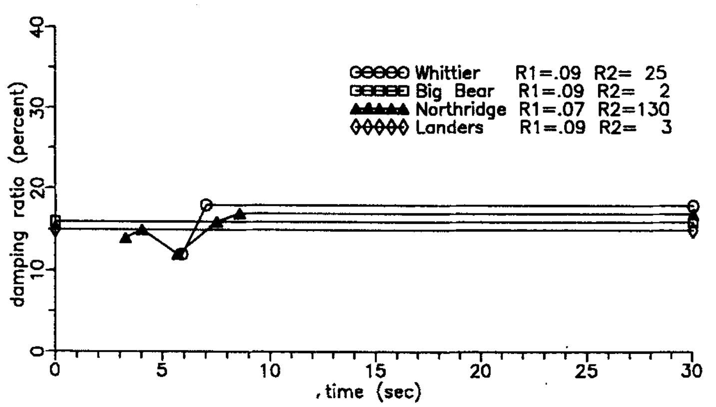


Figure 13. Comparison of the estimated system natural frequency and damping ratio of Van Nuys building from four different seismic response data

dynamics. The V value in Figure 12(a) is the loss function defined in equation (15) but with $\beta(t \cdot s) = 1{\cdot}0$. A smaller V value means larger number of time segments of adaptive gain is used. Figure 13 shows the comparisons of the estimated system natural frequency and damping ratio from these four earthquakes. The identified first mode natural frequency looks more reasonable than that obtained using the RLS algorithm. For the data of the Northridge earthquake, the parameters clearly show three regions. The initial part between 0 and 3 sec corresponds to linear behaviour of the original structure. The second part between 3 and 9 sec is where the damage takes place. The final from 9 sec and beyond corresponds to vibration of the damaged structure. The time variation of natural frequency and damping ratio from event to event looks more consistent and the time variation of the identified system parameters looks more reasonable. It is noticed that the identified system natural frequency and damping ratio from Lander and Big Bear earthquakes are time invariant. This observation suggests that the model gives a good estimate of the time variation of modal parameters, and the proposed selection of the parameters R_1 and R_2 by experience gives good results for the estimation of the modal frequency and damping ratio. The feature of the AFMM method is that if it detects a non-existent jump, it can correct the mistake without losing any valuable information, and the results of identification are much more reliable. Figure 14 shows the time variation of the modal parameters $A(q^{-1})$ from Northridge earthquake and Whittier earthquake data. Through adaptive forgetting in recursive identification the most reliable time-varying modal parameters can be obtained. Since the Northridge earthquake did induce significant seismic response of the building, the time variation of parameter $A(q^{-1})$ shows abrupt changes from time to time in the beginning of the response. The time-dependent transfer function of the building for Northridge earthquake is also shown in Figure 15.

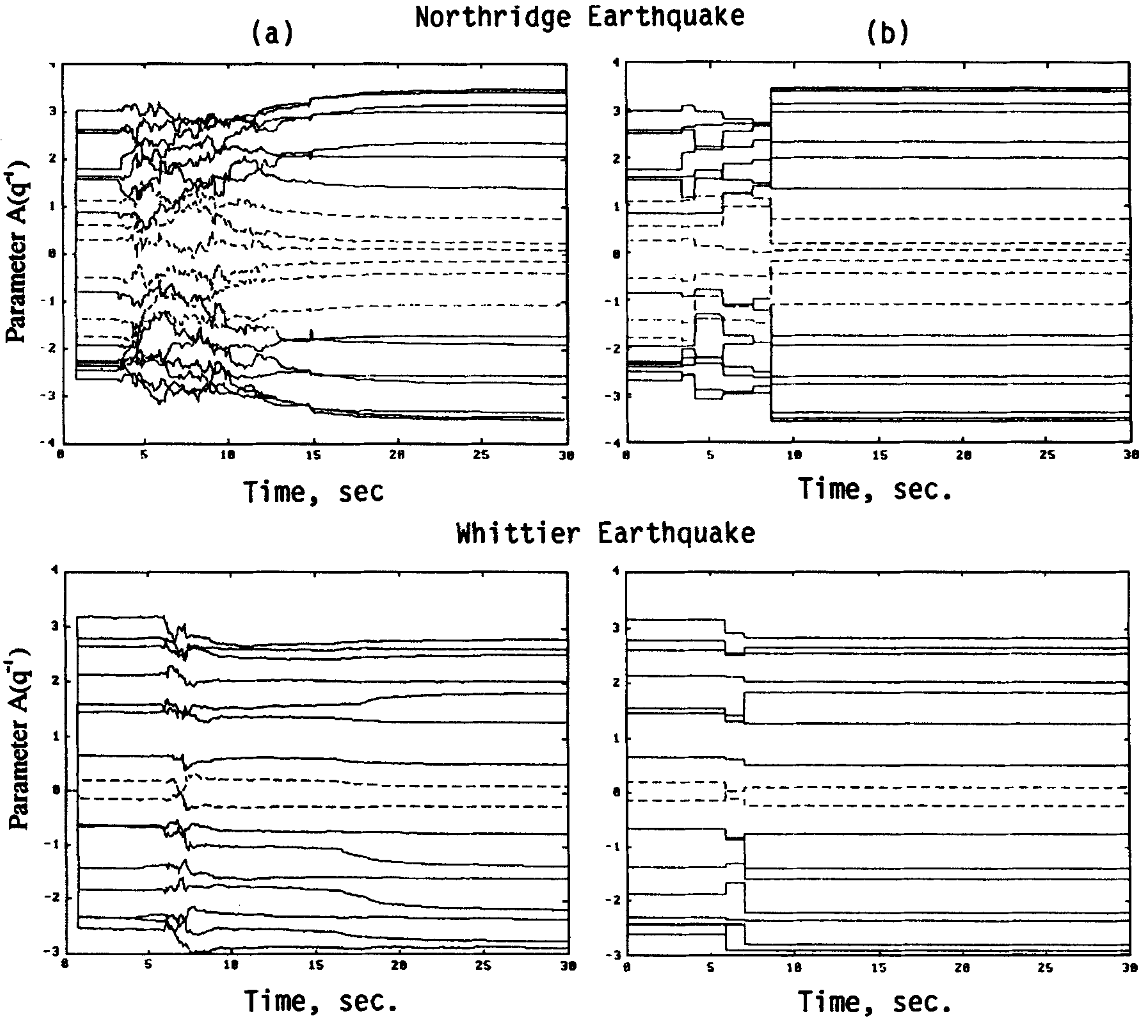


Figure 14. (a) Time variation of modal parameters, $A(q^{-1})$ in ARX model for Northridge and Whittier earthquakes. (b) Segments of parameter $A(q^{-1})$ corresponding to (a) through the AFMM method

CONCLUSIONS

The purpose of this paper is to discuss the discrete-time method for system identification based on linear filtering and recursive least-squares estimations. Under the assumption of a linear time-invariant system, the ARX and ARMAX models are used. Using the seismic response data of the Van Nuys seven-storey building, the modal frequencies, damping ratios and modal contributions are estimated. Emphasis was also placed on the time-varying modal parameters which were also estimated using the recursive least-squares method and adaptive forgetting through multiple models method. From the analysis of the recorded building response data, the following conclusions are drawn.

1. By increasing the order of the ARX model, closely spaced modes can be identified, and under such conditions the time-delay problem can also be neglected. On the contrary, the ARMAX model provides a system transfer function much smoother than the ARX model. For the off-line identification both models can be applied to structural system identification. But it provides only the behaviour of the equivalent linear system. For a system with non-linear response, the on-line identification must be used.

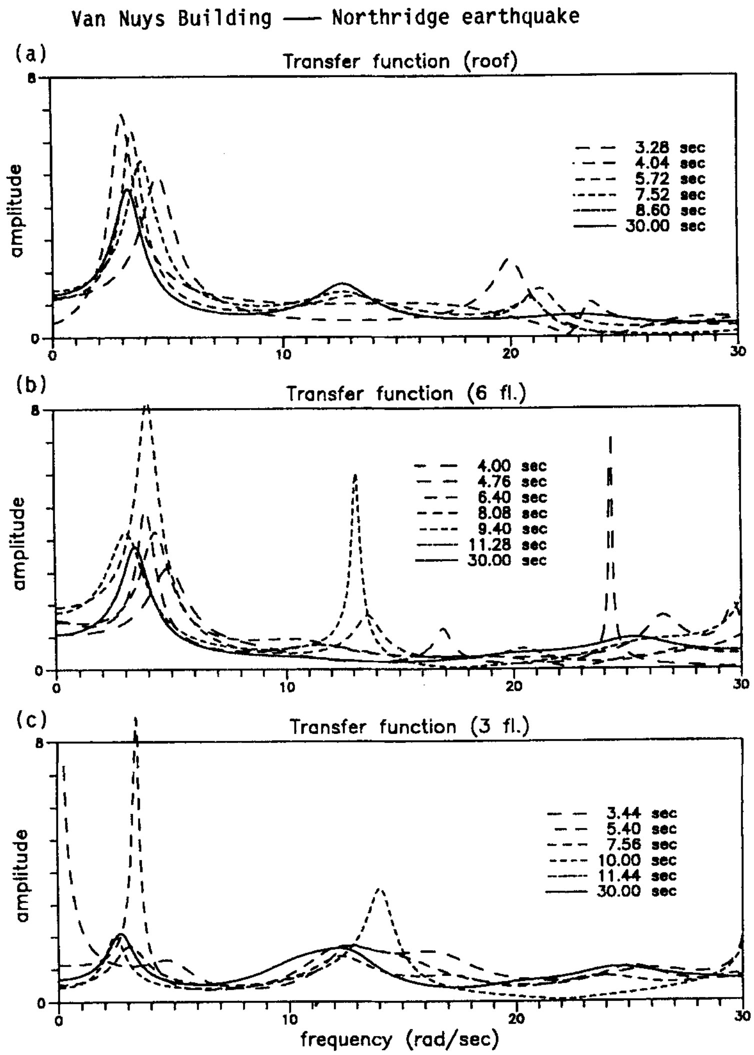


Figure 15. Estimated system transfer function at different times from the Van Nuys building (Northridge earthquake): (a) using roof response as output; (b) using 6th floor response as output; (c) using 3rd floor as output

2. Application of the recursive least-squares identification method is useful to the detection of time-varying modal parameters. The forgetting factor, the shape of which remains constant during the recursive process, is important for the estimation of modal parameters. A large gain (forgetting) means that it will quickly adapt to a new situation, but it will be sensitive to random error in the signal. Because of the constant shape in the forgetting factor, it cannot detect rapid changes of modal parameters.
3. The new recursive identification method, AFMM, is suited for identification of a system with rapidly changing parameters. The estimation results obtained by the RLS and AFMM methods are compared. The identified modal parameters (especially the structural damping ratio) look more reasonable for a system subjected to strong earthquake excitations if the AFMM method is used.
4. The AFMM method was applied to identify the time-variant modal parameters of the Van Nuys building subjected to four different earthquakes. The identified natural frequency and damping ratio from event to event are more reasonable than the results obtained using the recursive least-squares method. Since this building was damaged by the Northridge earthquake, the identified natural frequency and damping ratio from this seismic event show big differences from other events.

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